

## NOTES ON THE THEORETICAL DYNAMICS OF INTERMITTENT PUBLIC PASSENGER TRANSPORTATION SYSTEMS

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**Abstract**—This review paper examines the state of the art on the dynamics of vehicle pairing on bus, rapid-transit and elevator systems. Through examination of the limitations of analytic models of pairing, a description is developed of the dynamic interactions that produce vehicle pairing. Empirical evidence is presented to suggest that variability in vehicle journey time is a contributory factor. An evaluation of the consequences of pairing for passengers concludes that experiments investigating regulation of pairing should not only monitor passenger waiting times, but also measure passenger journey times and the degree of interdependence between these trip time components.

### INTRODUCTION

The last decade has seen a growing interest in the relationship between certain dynamic characteristics of intermittent public passenger transportation systems and their operational and economic efficiency. The systems concerned are urban bus transportation systems, multiple passenger elevator installations and urban tracked transportation systems. (The theoretical dynamics of tracked systems are slightly different from those of the other two systems in that overtaking is rarely feasible.) Prior research has centered on the phenomenon of vehicle pairing in these systems, on the derivation of optimal strategies to control pairing, and on the measurement of subsequent system performance.

This paper will review the prior literature on vehicle pairing dynamics. It is found that gross simplifications are required for the construction of analytical models of pairing dynamics. Various propositions are presented regarding those features which are neglected in analytical approaches, and consideration given to empirical evidence on the degree of their importance in determining vehicle journey time variance.

Examination of simulation models indicates that these additional factors which are presumed to influence pairing have not been generally investigated, since simulation has been almost invariably directed toward determination of optimal strategies to control pairing tendencies rather than towards the identification of the factors determining pairing and the evaluation of their relative importance.

The paper concludes with a review of the consequences of pairing, and describes how these should influence the design of future experiments with real or simulated intermittent public passenger transportation systems.

### ANALYTIC MODELS OF VEHICLE PAIRING

Newell and Potts (1964) are generally credited with being the first to describe the mechanism whereby buses tend to pair or "bunch" together. They describe how an initial disturbance to the scheduled running of

a vehicle may tend to intensify as the vehicle travels along a route, and propagate to successive vehicles due to the uneven accumulation of passengers. Alternate vehicles get progressively behind and ahead of schedule resulting in the pairing of vehicles on the route. Newell and Potts develop a mathematical model to describe these dynamics, and determine that the tendency to pair is theoretically a function of the ratio of the passenger arrival rate to loading rate.

Numerous authors have examined this and similar models. Potts and Tamlin (1964) attempted an empirical test of the model and found some support for the suggestion by Newell and Potts that bus pairing arises because of variation in the number of passengers boarding. Lehmann (1968) developed a model similar to that of Newell and Potts to explain deviation from scheduled running of trains. Sudmeyer (1969) criticised certain assumptions in Lehmann's model, assumptions which were subsequently defended by Vuchic (1969a). In another paper Vuchic (1969b) proposes some algebraic modifications to the Newell and Potts model.

Heap and Thomas (1976) emphasize the complexity of the dynamics of bunching, and suggest that past attempts at modelling even a simple service pattern had been highly idealized. Their reformulation of the Newell and Potts equation is as follows:

$$\tau_m(n) = \frac{(n+m-2)!}{(m-1)!(n-1)!} \left[ \frac{-\lambda T}{1-\lambda T} \right]^{m-1} \\ \times \left[ \frac{1}{1-\lambda T} \right]^{n-1} \tau_{(1)}(1).$$

Where  $\tau_m(n)$  is the delay of vehicle  $m$  at station stop  $n$ ;  $\lambda$  is the passenger arrival rate in passengers per second;  $T$  is the boarding time for one passenger.

Heap and Thomas describe their development of a modified model which contains a stochastic element to represent the random variation in stop time, but conclude that this model is valid only if the major cause of irregularity is variation in running times

rather than variation in loading times, and that their model breaks down if vehicles come too close to each other. (Since they were concerned with platooning tendencies of a tracked rapid-transit system, it was not feasible to assume great variation in running times. For bus systems however it appears that loading time variance is of the same order as running time variance, as noted below.) They then describe an alternative analytical method utilizing a Markov chain approach, but reject this method, concluding that in order to model the dynamics of a tracked system realistically, so as to represent such factors as passengers arriving at stops, the boarding process, and other stochastic variables, a simulation model must be used.

A similar sentiment is expressed by Osuna and Newell (1972), in one of the several papers which address the issue of mathematical derivation of optimal strategies for control of off-schedule running. They state that they were forced to abandon formal mathematics in favour of intuition in order to analyze even the limited problem of two vehicles. Although Newell (1974) did revert to a mathematical approach in a subsequent study, and was able to draw some broad conclusions from his intricate calculations, he was forced to admit that his theory had very limited practical application.

Most recently Chapman and Michel (1978) present an alternative mathematical model of the pairing mechanism which enables the deduction of the stop at which pairing will occur. As with most other authors, they conclude with the lament that whilst analytical models are simple and of general applicability, the simplifying assumptions necessary for the construction of such models are such as to raise doubts as to their ability to reproduce the dynamics of real systems.

#### THE LIMITATIONS OF ANALYTICAL MODELS

The Newell and Potts model presumes that the amplification of a delay to a vehicle and the propagation of schedule deviations to subsequent vehicles arises solely because of variation between successive vehicles in the time taken to board uneven numbers of passengers. This presumption has focussed attention away from determination of the existence and relative importance of other factors and feedback mechanisms which contribute to the tendency of vehicles to pair on intermittent public passenger transportation systems.

Various of the authors noted above do comment on the limitations of their modelling approach, but unfortunately do not elaborate on the consequences of these limitations. Newell and Potts note four factors which tend to dampen the delay amplification mechanism:—

(1) Buses are driven by drivers who can respond to the tendency to pair by adjusting vehicle speed. (This does not apply to driverless automatic systems. However this suggests an obvious way of minimizing the tendency to pair on automatic systems, provided the

system's normal operating speed is below the maximum achievable, and provided passengers could tolerate any increased acceleration and deceleration forces involved.)

(2) A delayed bus may become full, and may be able to gain lost time.

(3) The establishment of timing points along the route, and implementation of other controls can help partially stabilize the system.

(4) If overtaking is allowed, the tendency to pair becomes confused. (The subsequent literature indicates a general opinion that, in the absence of control mechanisms, paired vehicles remain in close proximity to each other, irrespective of allowing overtaking.)

These factors all seem to indicate that analytic models would tend to overestimate the degree of amplification of an initial deviation from schedule. Heap and Thomas point out a number of other deficiencies of such models:—

(1) Alighting time is ignored, and yet since the number of passengers on board vehicles in a disturbed system reflects the pattern of disturbance, alighting can itself amplify delays. A delayed vehicle will have a greater than average number of passengers wishing to alight at each stop. If alighting time exceeds boarding time at particular stops then the vehicle is further delayed.

(2) Passenger transfer time may be dependent on vehicle load, in that passengers may take longer to board and alight when the vehicle is almost full.

(3) The model relates to behaviour subsequent to a single disturbance. In real life there are many perturbations disrupting scheduled running, and the model is unable to deal with the modifications to behaviour of vehicles caused by delays subsequent to an initial delay. Whilst perturbations subsequent to an initial delay may often offset such a delay, the nature of traffic congestion is such that the probability of a vehicle being perturbed may increase with the amount of prior perturbation.

These three factors suggest that analytic models underestimate the degree of amplification of a delay to a vehicle. There are a number of other factors, not explicitly stated in the literature, which suggest further underestimation of the degree of delay amplification and propagation:

(1) Deterministic models treat passenger numbers as a continuous variable rather than as a discrete variable. Thus a bus may be stopping to pick up, say, 0.75 of a passenger at every stop. It is further assumed that each vehicle will halt at every stop on the route. However, if there is an average of 0.75 of a passenger at each stop, buses may in reality only halt to board a passenger at roughly 3 stops out of 4. Deterministic models may thus overestimate the number of stopping events required to pick up, and offload a given number of passengers.

This deficiency, which may have to be tolerated in certain applications and which indeed may be corrected by adjusting the specified average vehicle

speed, is apparent in a number of deterministic models of a bus route. This deficiency is particularly noticeable in analyses of optimal stop spacing, for example Lesley (1976). Wirasinghe and Ghonheim (1981) allow for the stochastic effect of discretionary stopping in their analysis, and suggest that this allows closer stop spacing. They do not explore any resultant effect on the propensity to pair produced by denser stop spacing. The simple model developed by Mohring (1971) also allows for discretionary stopping, but ignores any effect on service reliability. Vuchic (1969c) and Vuchic and Newell (1968) had previously considered stop spacing for rapid transit lines, but did not specify reliability as a measure of performance.

(2) The treatment of passenger numbers as a continuous variable, and the assumption that buses halt at every stop, results in the neglect of another major factor. With regular running (perfect adherence to schedule) and the same average passenger arrival rate at every stop, a certain number of stops will have no passengers waiting to board and no passengers wishing to alight. This will occur simply by virtue of the random nature of passenger arrivals. Cundill and Watts (1973) suggest that on average an urban bus will omit about one stop in six under these conditions.

Under idealised conditions of random passenger arrivals and equal average passenger arrival rates at each stop, a delay to a vehicle will increase the probability that it will have to halt at stops it would otherwise have bypassed. A longer than average interval between buses reduces the probability that stops will have no passenger waiting to board. Likewise for alighting, a delayed bus with a greater than average load will halt to offload passengers at a greater than average number of stops. A delayed bus will therefore experience a greater than average number of stopping events, as well as having to board and offload an above average number of passengers. The following bus will experience a lower than average number of stopping events, and deal with a lower than average number of passenger transfers.

The relaxation of the assumption of equal passenger arrival rates at all stops exacerbates this phenomenon. If, as on most real routes, there is a high proportion of low demand stops for which the probability of no passengers wishing to board or alight is appreciable, then the variation in the number of stopping events between successive buses will be high, and will substantially contribute to the dynamics of delay amplification.

Clearly this does not apply to tracked transportation systems for which there are no discretionary stops. However for elevator systems stopping event variability is obviously of greater importance than for bus systems. Since stopping events for elevators may be more lengthy than inter-floor travel times, passengers may be acutely aware of this phenomenon.

(3) Newell and Potts note that bus drivers can correct the tendency to pair by adjusting vehicle

speed. The adjustment may be in the opposite direction however, for it would appear that pairing dynamics offer an opportunity for astute drivers to get ahead of schedule and reduce their workload and on-route time. Abuse of this opportunity to run "sharp" may however produce formal or informal sanctions on the drivers concerned.

On bus systems the near universal adoption of one-person operation may have reinforced tendencies to run sharp. Drivers now have to interact with passengers, and experience the problems of getting behind schedule at first hand. Driver skill may have a part to play in this process also. Experienced employees may be both skillful in their driving techniques and in the speed with which they are able to board passengers. Novitiate employees on the other hand may not possess these capabilities, and their problems may be compounded by the ability of experienced drivers to avoid getting behind schedule. This reasoning suggests that delayed buses may be found to be more likely driven by less experienced or less proficient drivers. Variation in driver skill and experience may thus compound the amplification of delays.

(4) Himanen (1975) has noted that heavily loaded vehicles have lower average running speeds than lightly loaded vehicles because of the slower acceleration and deceleration of heavier vehicles. This would tend to amplify disturbances since delayed buses are usually more heavily loaded.

(5) Variations in short-term local traffic conditions (as opposed to longer-term variation between peak and off-peak conditions) might be expected to randomly reduce or augment pairing tendencies. This may not be the case however, as any variation in traffic conditions is unlikely to separate paired vehicles, but is likely to affect well-separated vehicles differently, so producing a variation in headway that augments the pairing process. If the degree of correlation of the link travel times of two successive buses is dependent on the interval between them then short-term variation in local traffic conditions will reinforce pairing tendencies.

(6) Weak dispatch timing management and inadequate layover provisions may allow the schedule deviations which accumulate for buses on a route to affect their next journey along the route. Allowing an early arriving bus to be dispatched early, or being forced to allow a late arriving bus to layover past its normal dispatch time enables the schedule deviations from one journey to influence the next journey schedule of the bus.

The above descriptions indicate the dynamics of pairing on transportation systems to be an immensely complex process. It is not surprising that past researchers have had to resort to gross simplification in order to make the problem amenable to mathematical analysis. The simplification required for the application of this methodology would appear to be so gross however as to raise doubts as to the degree to which a complete understanding of the pairing

phenomenon has been developed through the use of mathematical analysis.

**VEHICLE JOURNEY TIME VARIANCE AND PAIRING DYNAMICS**

The simple feedback model utilized by Newell and Potts and others presumes that pairing arises solely because of variation between consecutive vehicles in the number of passengers boarding at each stop, and that the degree of feedback is a function of the ratio of passenger arrival rate to boarding rate. This ratio is clearly a key determinant of the extent to which schedule perturbations are amplified.

The propositions outlined in the preceding section indicate complex additional allied feedback mechanisms. These are illustrated in Fig. 1, which shows some of the contributory sources of variation in the journey time of a bus along a hypothetical route. Also shown are the linkages between these sources which provide the mechanisms through which pairing arises.

If Fig. 1 is a more complete graphical representation of the dynamics of pairing, then it is of interest to attempt to ascertain the relative importance of the various factors involved. In particular it appears

relevant to determine the contribution to vehicle journey time variance caused by variation between consecutive vehicles in the number of stopping events along a route. This variance may well be of the same order as the contribution caused by variation in the time taken to board differing numbers of passengers at stops, the previously presumed sole mechanism behind the pairing phenomenon. (Any intuitive consideration of the nature of pairing on elevator systems would immediately indicate that variation in the number of stopping events between consecutive vehicles provides the main source of variation contributing to elevator journey time variance.)

Some clues can be obtained from empirical data, specifically the data presented by Chapman *et al.* (1976). Table 1 shows their analysis of sources of contribution to the variance in bus journey time on Newcastle-upon-Tyne route 33.

The variances shown in Table 1 are estimates derived from measurements taken on the route, and are those that would theoretically occur over a typical link on the relevant section of the route. The authors note that the variance of total journey time on the link would be equal to the sum of the variance of the components of the journey time if the sources of var-

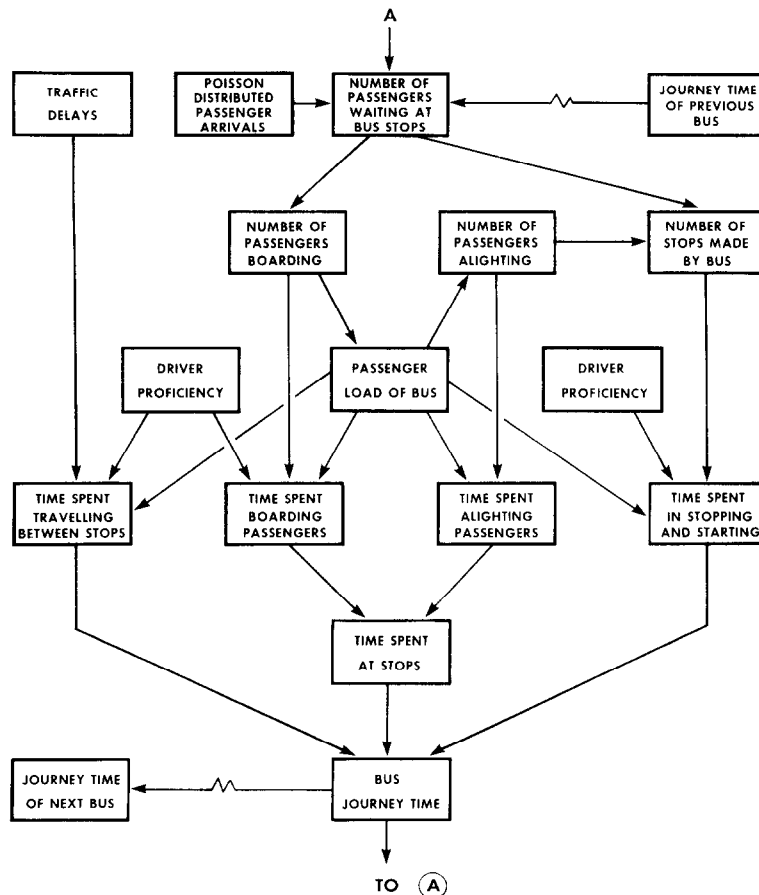


Fig. 1. Sources of variation in the journey time of a bus along a hypothetical route.

Table 1. Contribution to the variance in bus journey time on Newcastle-upon-Tyne route 33

Source of variation contributing to the variance in journey time	Contribution to the Variance		
	inside the city centre (seconds <sup>2</sup> )	outside the city centre in bound (seconds <sup>2</sup> )	outside the city centre outbound (seconds <sup>2</sup> )
Probability of buses stopping at stops	100	160	100
Different numbers of passengers wishing to board	50	60	20
Different boarding times of passengers	70	80	40
Different numbers of passengers wishing to alight	10	5	5
Different alighting times of passengers	20	20	20
Dead times	5	5	5
Different travel times between stops*	680	110	170
Different time spent in queue delays*	(400)	(40)	(40)

\*Since queue delay is a component of travel time, part of the variation in travel time is due to variation in queue delay.

iation were independent of each other. Of course, the pairing phenomenon implies interdependencies between the component variances and increases in the sizes of these variances resulting from these interdependencies.

Despite this objection to the data, Table 1 may provide some insights. The variance attributable to the probability of buses stopping at stops ( $p = 0.9$  inside the city centre,  $p = 0.8$  outside the centre) is substantial, and apparently of the same order as the variances associated with boarding and alighting passengers. This would indicate stopping event variability to be a major factor in pairing dynamics.

What Table 1 does not indicate is how the component variances increase when system performance deteriorates through pairing. In essence the variances shown indicate the importance of the various factors in dictating the initial deviation from schedule of buses with perfectly even headways. As soon as a schedule deviation occurs, the interdependencies shown in Fig. 1 take effect, and certain of the variances will increase. If there were any way of determining these rates of increase, the first derivatives of the relevant variances as vehicles begin to pair, then it would be easier to determine the relative importance of the various factors.

Some further evidence of journey time variance on a real route, where pairing was an observed phenomenon, is given by Chapman *et al.* Table 2 gives the actual measurements of components of journey time on the full 8 km length of Newcastle-upon-Tyne route 33.

These data do indeed indicate that variance in the time spent travelling between stops (which includes variance in the number of stopping events) is of the same order as the variance of time spent at stops. However, the data neither indicate the degree of

Table 2. Measurements of components of bus journey time on Newcastle-upon-Tyne route 33

Components of journey time	Mean (minutes)	Standard Deviation (minutes)	Variance (minutes <sup>2</sup> )
Time at bus stops	7.9	3.3	10.9
Time travelling between bus stops	22.6	3.5	12.3

pairing on the route, which would be reflected by interdependence between the tabulated distributions, nor do they enable substantial clarification of the relative importance of the individual factors determining the extent of pairing. As a consequence, the empirical evidence presented by Chapman *et al.*, whilst being the most extensive of its kind, is of limited usefulness in understanding the dynamic relationships portrayed in Fig. 1. Such an understanding would seem to require some form of sensitivity analysis, an investigative technique most likely to be found in simulation approaches to the study of transportation systems.

#### INVESTIGATION OF PAIRING DYNAMICS THROUGH SIMULATION

With the exception of Gamse (1976) who describes a simulation model of an elevator system, all other simulations of individual vehicle movements are of bus systems. Six U.K. simulation models and one French simulation are reviewed by Jenkins (1976). Other U.K. simulations are reported by Wren (1971), Jackson (1976) (1977a) (1977b), Jackson *et al.* (1977), and Boyd (1978). Simulations of U.S. systems are reported by Erikson (1972), Radelat (1973), Lavin (1974), Trygar (1974), Koffman (1978), Magro and Huttinger (1980), Turnquist and Bowman (1980), and Turnquist (1981).

Examination of these various simulation studies is disappointing however, in that the majority of those which involve consideration of pairing dynamics do not so much focus on the causes of the phenomenon but rather investigate strategies to regulate its effects. This is a surprising emphasis, given the limitations of mathematical models and the consequent advocacy of a simulation approach by several of the mathematical analysts.

Simulation models have enabled the identification of some of the individual components and linkages involved in the pairing mechanism. Even the simplest simulation models demonstrate the tendency of buses to pair. Lesley and Jackson *et al.*, have demonstrated that boarding time is a key determinant of delay amplification. Boyd has demonstrated how demand level is also a determining factor, as additionally predicted by Newell and Potts.

The most complete investigation of the factors affecting pairing was conducted by Bly and Jackson

(1974). Their model of a Bristol bus route was used to investigate the relative importance of the variations in time spent by buses at stops and between stops and the variations caused by poor time-keeping at terminals. Each of these three types of variation was removed in turn and the effect on average passenger waiting time calculated in each case, producing the results shown in Table 3.

The results indicate the elimination of variation in time spent between stops to result in the greatest improvement in service reliability, as measured by reduction in mean passenger waiting time. However, as with the empirical data presented by Chapman *et al.* this sensitivity analysis conducted by Bly and Jackson is confounded by the interdependencies inherent in the dynamics of pairing. The experiment is unable to distinguish cause and effect in the interdependence between variation in the time spent at stops and variation in the time spent between stops, and hence the results are of reduced value.

The work of Bly and Jackson does suggest though that, for example, the provision of bus-only traffic lanes as a means of reducing between-stop travel time variance may be a feasible means of stabilising pairing. Another solution may be the wider spacing of bus stops so as to decrease the probability that buses bypass stops, though of course the benefits produced would be diminished by the cost of increased passenger access time. These solutions are further discussed in Turnquist (1981). In contrast the analytic models that follow from Newell and Potts would not suggest these as feasible solutions, but rather solve the problems of pairing through the provision of buffers within the system.

The simulation by Turnquist and Bowman (1980) confirms that link travel time variability contributes to vehicle pairing. They further note that an increase in vehicle frequency for a given level of demand also contributes to vehicle pairing and produces a deterioration in relative reliability as measured by the coefficient of variation of vehicle arrival times. Their model also confirms that pairing serves to limit the extent to which service can deteriorate, in that once vehicle pairs have formed (at increasingly earlier positions on a route as vehicle frequency increases), the system appears to stabilize. (This raises the incidental question of the possible existence of instability arising from second-order pairing dynamics.

Whilst of minor importance for most bus systems, second-order pairing leading to the formation of groups of four or more vehicles may be of relevance to the dynamics of multiple elevator systems.)

#### THE CONSEQUENCES OF PAIRING

We may examine the consequences for consumers of increased pairing by considering all passengers who board at a specific stop and travel to a specific destination. Increased pairing implies increased vehicle headway variance at the boarding stop. This produces an increase in average passenger waiting time and increased waiting time variance. These consequences are well known, and indeed most simulation models concerned with derivation of optimal strategies to dampen pairing specify minimization of average waiting time and the reduction of extreme waiting times as objective functions.

Some studies, for example Turnquist (1981), have identified in-vehicle travel time as an additional measure of performance, reflecting the relationship between link travel time and unreliability. These studies use vehicle travel time as a surrogate measure of passenger in-vehicle travel time, and consequently overlook an important theoretical effect of increased pairing on the in-vehicle journey times of passengers. Increased pairing, produced by, say, an increase in average passenger boarding time, must increase the variance in vehicle passenger loads and increase the variance in overall vehicle speeds. (This latter effect arises because of increased variability in the number of passengers boarding and alighting vehicles and in the number of stopping events experienced by each vehicle.) There is negative co-variance between vehicle passenger loads and vehicle speeds such that, e.g. delayed vehicles have higher passenger loads than before and travel at reduced average speeds when pairing increases. This results in an increase in average in-vehicle time for passengers and an increase in the variance of journey time.

How large these increases are is open to question, there never having been a study to address the issue. Intuitively, for elevator systems these increases in in-vehicle time may be quite large in comparison to the corresponding increases in waiting time that result from increased elevator bunching.

One important consequence of this feature of pairing is that on disturbed systems overall average passenger travel speed will be lower than overall average vehicle speed because there are larger numbers of passengers travelling on the slower vehicles than there are on the faster vehicles. The use of vehicle travel time as a surrogate measure of passenger in-vehicle travel time would thus produce an underestimation of passenger travel time in disturbed systems. For similar reasons, the degree of uncertainty of passenger arrival times at the destination stop will be likely underestimated if monitored merely by measurement of variance in vehicle arrival times at the stop.

Table 3. Average passenger waiting time on Bristol route 9

Variations excluded in the model	Average passenger waiting time (minutes)	Reduction in average waiting time (minutes)
None	6.18	—
Poor timekeeping at terminals	5.93	0.25
Time spent between bus stops	5.35	0.83
Time spent at bus stops	5.73	0.45

A final consequence for passengers may be that increased pairing produces an increase in the positive co-variance between waiting time and in-vehicle journey time. This elimination of any statistical independence of waiting time and journey time by virtue of pairing theoretically arises as follows: the longer a passenger waits at a stop, the higher becomes the probability that the next vehicle to arrive at the stop will be a delayed vehicle rather than a vehicle ahead of schedule. Since a journey on a delayed vehicle takes longer than a journey on a vehicle ahead of schedule, then longer than average waits will tend to be associated with longer than average journey times. Likewise, short waits will tend to be followed by shorter journey times to the same destination.

Quite clearly such a positive co-variance, if it is associated with pairing, would be an extreme irritation for passengers. This may be a plausible and hitherto unexplained reinforcing reason why surveys of passenger attitudes have repeatedly found the reliability of scheduled arrival times to be one of the most important desired characteristics of transit systems, although it is likely that this co-variance may be of secondary importance because of the difficulty of an individual detecting such a relationship through casual observation.

#### DISCUSSION

The pairing of vehicles on intermittent public passenger transportation systems is a complex yet fascinating empirical phenomenon. The use of mathematics as a means of understanding the mechanisms involved has been of limited success, this analytical method being unable to cope with the full complexity of the system's dynamics. Through an intuitive approach, various propositions concerning hitherto unidentified aspects of the dynamics of pairing have been outlined in this paper.

Whilst empirical evidence would appear to confirm the existence of both the previously confirmed and the proposed additional mechanisms of pairing, the full identification of the relative importance of the various factors involved must await a comprehensive sensitivity analysis conducted through simulation. Such an analysis may yield insights into the dynamics of pairing that lead to novel solutions to the problems posed by systematic schedule instability.

For any such sensitivity analysis, and indeed for any further investigation of optimal control strategies through the use of simulation, some consideration should be given to the respecification of measures of system performance. If, as theorized, pairing affects passengers' in-vehicle journey times and induces positive co-variance between waiting time and journey time, then future experiments should not only monitor effects on passenger waiting times, but also include measurements of these parameters.

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#### REFERENCES

- Bly P. H. and Jackson R. L. (1974) Evaluation of Bus Control Strategies by Simulation. Transport and Road Research Laboratory, *Report LR637*, Crowthorne, England.
- Boyd C. W. (1978) One-man operation of urban bus routes: System dynamics, costs and benefits. Ph.D. Thesis, Cranfield Institute of Technology, U.K.
- Chapman R. A., Gault H. E. and Jenkins I. A. (1976) Factors affecting the operation of urban bus routes. *Working Paper No. 23*, Transport Operations Group, University of Newcastle.
- Chapman R. A. and Michel J. F. (1978) Modelling the tendency of buses to form pairs. *Transpn Sci.* **12**(2), 165–175.
- Cundill M. A. and Watts P. F. (1973) Bus boarding and alighting times. Transport and Road Research Laboratory, *Report LR521*, Crowthorne, England.
- Eriksen A. R. (1972) A central area transit simulation model. *Proc. 7th Annual A.C.M. Urban Symposium On the Application of Computers to Problems of Urban Society*, A.C.M., New York.
- Ganse B. (1976) A simulation of an elevator system for a moderate height building. *Special Report UCB-ITS-SR-76-5*, Institute for Transport Studies, University of California, Berkeley.
- Heap R. C. and Thomas T. H. (1976) The modelling of platooning tendencies in public transport. *Traff. Engng. & Control* **17**(8/9), 360–362.
- Himanen V. (1975) Journey time and vehicle occupancy characteristics of the Helsinki metropolitan public transport system. Research Paper 55, Helsinki University of Technology.
- Jackson R. L. (1976) An evaluation of bus control systems by simulation. Transport and Road Research Laboratory, *Report LR712*, Crowthorne, England.
- Jackson R. L. (1977a) Evaluation by simulation of control strategies for a high frequency bus service. Transport and Road Research Laboratory, *Report LR807*, Crowthorne, England.
- Jackson R. L. (1977b) Simulation studies of bus operations and control on London transport route 11. Transport and Road Research Laboratory, Supplementary Report SR339, Crowthorne, England.
- Jackson R. L., Coe G. A. and Finnamore A. J. (1977) A study of some factors affecting bus service performance. Transport and Road Research Laboratory, *Report LR767*, Crowthorne, England.
- Jenkins I. A. (1976) A comparison of several techniques for simulating bus routes. *Working Paper No. 14*, Transport Operations Research Group, University of Newcastle.
- Koffman D. (1978) A simulation of alternative real-time bus headway control strategies. *Transpn Res. Rec.* **633**, 41–46.
- Lavin O. W. (1974) Use of computer simulation for analysis of communications requirements. *Paper 149, I.E.E.E. Int. Conf. Communication*, Minneapolis (June 1974).
- Lehmann H. (1968) Verkehrliche Auswirkungen der Verspatungen. *Verkehr Tech.* **21**(3), 57–58.
- Lesley L. J. S. (1975) The role of the timetable in maintaining bus service reliability. *Proc. Symp. Operating Public Transport*, University of Newcastle.
- Lesley L. J. S. (1976) Optimum bus stop spacing: Parts 1 and 2. *Traff. Engng Control* **17**(10), 399–401 and **17**(11), 472–475.
- Magro W. and Huttinger J. L. (1980) A model to determine operational efficiency of transit vehicles on simulated bus

- routes. *TIMS/ORSA Conf.*, Washington D.C. (May 1980).
- Mohring H. (1972) Optimisation and scale economies in urban bus transportation. *Am. Econ. Rev.* **62**(3), 591–604.
- Newell G. F. (1974) Control of pairing of vehicles on a public transportation route: two vehicles, one control point. *Transpn Sci.* **8**(3), 248–264.
- Newell G. F. and Potts R. B. (1964) Maintaining a bus schedule. *Aust. Road Res.* **2**(1), 388–393.
- Osuna E. E. and Newell G. F. (1972) Control strategies for an idealized public transportation system. *Transpn Sci.* **6**(1), 52–72.
- Potts R. B. and Tamlin T. H. (1964) Pairing of buses. *Aust. Road Res.* **2**(2), 3–9.
- Radelat G. (1973) Simulation of urban bus operation on signalized arterials. Report FHWA RD-74 6 252P, Traffic Systems Division, Federal Highway Administration, Washington, D.C.
- Sudmeyer E. (1969) Nochmals: Verkehrliche Auswirkungen der Verspätungen. *Verkehr Tech.* **21**(10), 272.
- Tryger T. A. (1974) Two stochastic simulators for testing fixed route and demand actuated transportation systems. Ph.D. Thesis, Dept. of Electrical Engineering, Pittsburgh University.
- Turnquist M. A. and Bowman L. A. (1980) The effects of network structure of reliability of transit service. *Transpn Res.* **14B**, 79–86.
- Turnquist M. A. (1981) Strategies for improving reliability of bus transit services. *Transpn Res. Rec.* **818**, 7–13.
- Vuchic V. R. and Newell G. F. (1968) Rapid transit interstation spacing for minimum travel time. *Transpn Sci.* **2**(3), 303–339.
- Vuchic V. R. (1969a) Fahrplanstörungen—zusätzliche Faktoren und betriebliche Massnahmen. *Verkehr Tech.* **22**(4), 97–100.
- Vuchic V. R. (1969b) Propagation of schedule disturbances in line-haul passenger transportation. *Revue Un. Int. Trans. Publ.*, **XVII** 4, 279–284.
- Wirasinghe S. C. and Ghoneim N. S. (1981) Spacing of bus-stops for many to many travel demand. *Transpn Sci.* **15**, 3, 210–221.
- Wren A. (1971) *Computers in Transport Planning and Operation*, Ian Allen, London.