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# FARE EVASION AND NON-COMPLIANCE

## *A Simple Model*

By Colin Boyd, Christine Martini,  
John Rickard and Allen Russell\*

Interest in the economics and administration of ticket inspection within public transport systems has been sparked by the need to improve labour productivity, in the face of competition between modes and the growth of car ownership. Railways, for example, have typically reduced labour costs by substituting random ticket inspection for the previous total audit of all tickets, either at entry barriers or on board the train. On bus and tram systems in Continental Europe it is now common to find an honour system of self-cancellation of tickets by passengers and random ticket inspection by the transport operator. The extensive promotion of multi-ride tickets and the use of kerbside automatic ticket machines have reduced ticket-issuing costs, while the elimination of ticket inspection at point of entry has made possible the deployment of multiple-entrance articulated vehicles with large passenger capacities.

The adoption of European-style honour ticketing schemes by North American transit operators has been described by Diebel (1981) and Fox (1982). A major difficulty is the increased opportunity for fare evasion under the honour regime. Commenting on the prospects for the introduction of self-ticketing by passengers in the Portland Tri-County Transportation District of Oregon, Diebel (1981, p. 56) stated that:

Lost revenues due to fare evasion are, indeed, a real concern. This is why self-service must have a viable enforcement program – to keep the normally honest passenger from the temptation to cheat the system.

The initial start and the subsequent failure of the Portland experiment were reported as follows:

As in other cities with honour fares, inspectors randomly checked passengers to enforce the program. Tri-Met officials initially estimated . . . (that) over \$1.5 million in operating costs would be saved annually, due largely to short

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dwell times and better schedule adherence. Concerns over the high cost of administering self-service fare collection along with growing tensions between riders and inspectors, however, led to Portland's scuttling of the honour fare program in June 1984. During the nearly two years of implementation, Portland's Tri-Met experienced fare evasion rates of 5% to 6%, resulting in monthly revenue losses over \$100,000 (Cervero, 1985, p. 127).

Clearly, a move from a full audit ticket inspection system to a spot-check system calls for some appreciable redeployment of labour, and a revised set of labour management skills. One difficulty, illustrated by the Portland experience, is how to determine the optimal level of inspection resource so as to counter loss of revenue from fare evasion. The ticket inspection problem just described is a particular example of the wider problem of evasion and non-compliance, which we now attempt to model.

The model is built up from the already well-established mathematical literature on tax evasion: see, for example, Allingham and Sandmo (1972), Singh (1973), Srinivasan (1973), Yitzhaki (1974), Rickard, Russell and Howroyd (1982) and Russell and Rickard (1987). However it is strongly motivated by the behavioural literature on the subject: for example, Wallschutzky (1985a, b), Spicer and Hero (1985). Some further theoretical issues related to enforcement are discussed in Boyd (1986). The model takes into account both the perceived probability of detection and the actual probability of detection, as influenced by the level of expenditure on inspection.

### THE MODEL

Consider an individual,  $i$ , confronted with the opportunity to evade payment for various kinds of goods or services. The supplier of the goods or services, be it a company, government authority or whatever, sets a level of inspection which determines  $\pi$ , the actual probability of detection;  $\pi$  is some function of the inspection level  $x$ ,  $\pi = \pi(x)$ , and will apply in some specified time period.

The individual has some *ex-ante perception* of the probability of detection, which is dependent upon the efficiency of inspection. It is this perceived probability of detection which influences the individual's behaviour when confronted with the choice of fee payment or evasion.

Suppose the individual,  $i$ , has perceived probability,  $\pi_i$ , of being detected, with  $f$  the fee for service per trip, unit time, or whatever appropriate. Let  $p$  be the penalty (inclusive of recovery of amount unpaid) if he is caught evading the fee. The payoffs consequent upon the decision to evade ( $E$ ) or not to evade ( $NE$ ) are given as solutions to the decision problem in Figure 1.

The expected monetary value ( $EMV$ ) of the decisions to evade and not to evade are given by:

$$EMV(E) = -\pi_i p,$$

and

$$EMV(NE) = -f.$$

	D	ND
	$p(D) = \pi_i$	$p(ND) = 1 - \pi_i$
E	-p	0
NE	-f	-f

FIGURE 1

*The Basic Decision Problem for an Individual, i, where D and ND Denote the States of Nature Detected and Not Detected, Respectively*

The individual will choose to evade if the perceived probability of detection is such that  $EMV(E) > EMV(NE)$ , that is

$$-\pi_i p > -f,$$

or

$$\pi_i < f/p. \tag{1}$$

Here we have used expected monetary value, rather than a utility function, so there is the implied assumption of risk neutrality. For many relevant systems the fee and, for that matter, the penalty are relatively small, so the assumption of risk neutrality may not be unreasonable. It is also assumed that  $p$  (and  $f$  of course!) are known to all with certainty. For this particular analysis  $f$  and  $p$  are fixed, and  $x$ , the inspection level, is varied. In subsequent contributions we intend to investigate situations in which various combinations of  $f$ ,  $p$  and  $x$  are simultaneously chosen by the authority as part of its strategy.

### THE LINEAR CASE

Having set up this general model, we will now make some specific assumptions about the shape of the function  $\pi(x)$  and the distribution of  $\pi_i$  across individuals. Our intention here is to investigate the model structure and the interactions taking place within it. For the present, we are not overly concerned with the reality of our assumed functional form for  $\pi(x)$ . The simplest form for  $\pi(x)$  is a linear one, say  $\pi = kx$ , as depicted in Figure 2. This form could be applicable in a transport system where the actual probability of detection might be directly proportional to the number of inspectors employed, assuming that there are “sufficient” evaders and no crowding of inspectors.

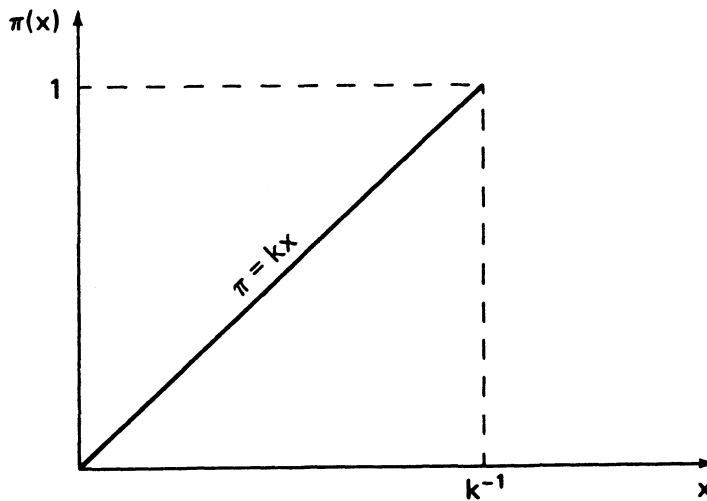


FIGURE 2  
A Linear Form for  $\pi(x)$

The distribution of  $\pi_i$ 's is more difficult. Intuition suggests a distribution such as the triangular distribution shown in Figure 3 with the apex located at  $\pi(x)$ , the *actual* probability of detection. Again, we emphasise that we are *not* arguing that  $\pi_i$  will be of this form, but rather that it is a plausible model to examine in the first instance.

Assume now that some individuals are always honest, irrespective of the level of inspection. Let  $\alpha$  be the number of honest individuals in population size  $n$ . The profit  $P$  can be measured as the revenue from the operation together with any revenue from penalties, less any costs incurred.

The authority faced with the problem of how many inspectors to employ may seek to maximise the profit  $P$ . Alternatively, it may set out to ensure that the profit is at least as much as it would be if the population were totally honest. We will now explore each of these inspection-resource decisions, given the linear form for  $\pi(x)$  as in Figure 2 and  $\pi_i$  distributed as in Figure 3.

Suppose that  $f/p < \pi(x)$ , that is, the ratio of fee to penalty is less than the actual probability of detection for a given level of expenditure  $x$ . This is not an unreasonable assumption; for example, if the penalty,  $p$ , were twice the amount of the fee, this would require  $\pi(x) > 1/2$ . If, as is the case in many public transport systems, the penalty for travelling without a valid ticket is more in the order of 50 to 100 times the value of the ticket, then the assumption is even more reasonable.

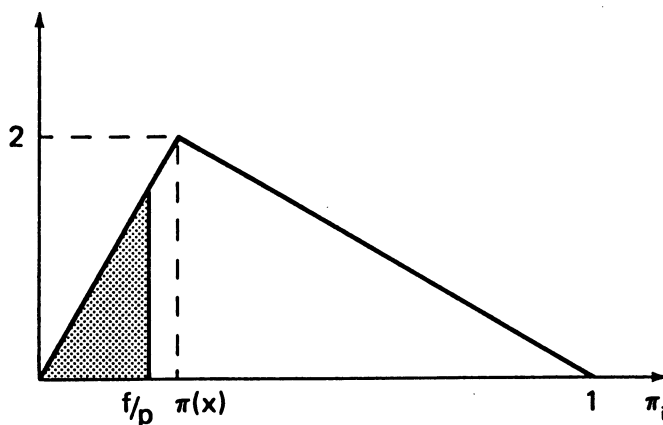


FIGURE 3

*Probability Distribution for  $\pi_i$*

In this case the proportion of potential evaders who decide to evade is given by

$$\frac{f^2}{p^2} \frac{1}{kx}$$

and is illustrated by the shaded area in Figure 3.

The profit  $P$  reduces to

$$P = nf + \frac{(n - \alpha)f^2}{p^2} \left[ p - \frac{f}{kx} \right] - (F_1 + cx), \tag{2}$$

where  $F_1$  is the fixed cost and  $c$  represents the marginal cost of inspection. As stated previously, we are interested in the variation of  $P$  with respect to the level of inspection  $x$ . Suppose that the authority seeks to set an inspection level  $x$  in order to maximise the profit  $P$ , given that the fee and penalty levels are fixed. It follows from (2) that

$$x = x^* = \frac{1}{p} \left[ \frac{(n - \alpha)f^3}{ck} \right]^{1/2} \tag{3}$$

and it is readily demonstrated that  $d^2P/dx^2 < 0$  when  $x = x^*$ . It follows that  $P$  is maximised at  $x = x^*$ . This is, of course, the level of inspection at which marginal (fee plus penalty) revenue is equal to marginal cost.

For the purposes of illustration, consider a simplified transport system in which there are 20,000 commuters per day, together with a team of inspectors, each of whom can inspect 400 tickets per day. We assume a uniform ticket cost of £1 and a penalty of £40 for being detected without a ticket. Ignoring inefficiencies arising from variation of travel time, length of journey, scheduling of inspectors,

duplication of inspection, etc., we take

$$k = \frac{400}{20,000} = \frac{1}{50} .$$

Assuming further that 30 per cent of commuters are perpetually honest, and that the salary of an inspector is £40 per day, it follows from (3) that the optimal number of inspectors is 6. This, in turn, implies an inspection rate of 12 per cent. The inefficiencies referred to above would in practice be expected to reduce  $k$ , and so to increase the number of inspectors.

Suppose now that we set the inspection level so that the profit from the operation will be equal to the profit that would be generated if the population were totally honest. The condition for this is

$$nf + \frac{(n - \alpha)f^2}{p^2} \left( p - \frac{f}{kx} \right) - (F_1 + cx) = nf - F_2, \tag{4}$$

where, in this case,  $F_1$  represents the fixed costs associated with the operation and  $F_2$  the fixed costs which would be incurred if all the population were honest.

Equation (4) reduces to a quadratic in  $x$ , from which it is readily shown that

$$2cx = \frac{(n - \alpha)f^2}{p} - F \pm \left[ \left( \frac{(n - \alpha)f^2}{p} - F \right)^2 - \frac{4c(n - \alpha)f^3}{p^2} \right]^{1/2}, \tag{5}$$

where  $F = F_1 - F_2$ .

Thus there are two levels of inspection that will ensure that the profit from the operation is equal to that which would be generated in a totally honest population. The higher level obviously generates more penalty revenue than the lower level, but this increased penalty revenue is offset by additional costs of inspection.

The graph of  $P$  against  $x$ , for  $x > 0$ , is illustrated in Figure 4. Clearly  $x^*$  is the profit maximising inspection level, while  $x_2$  and  $x_3$  are the solutions to equation (5);  $x_1$  and  $x_4$  are obtained by solving  $P = 0$ . For  $x_2 < x < x_3$  (that is, in the shaded region of Figure 4) the profit generated is higher than would be generated if the population were totally honest.

We now investigate the sensitivity of this maximum attainable profit,  $P^*$ , to changes in the other parameter values. It will be convenient to write  $P^*$  in the form

$$P^* = nf + \frac{(n - \alpha)f^2}{p} - \frac{2}{p} \left( \frac{(n - \alpha)f^3 c}{k} \right)^{1/2} - F_1,$$

from which it readily follows that

$$\frac{\partial P^*}{\partial p} = p^{-2} \left[ 2 \left( \frac{(n - \alpha)f^3 c}{k} \right)^{1/2} - (n - \alpha)f^2 \right].$$

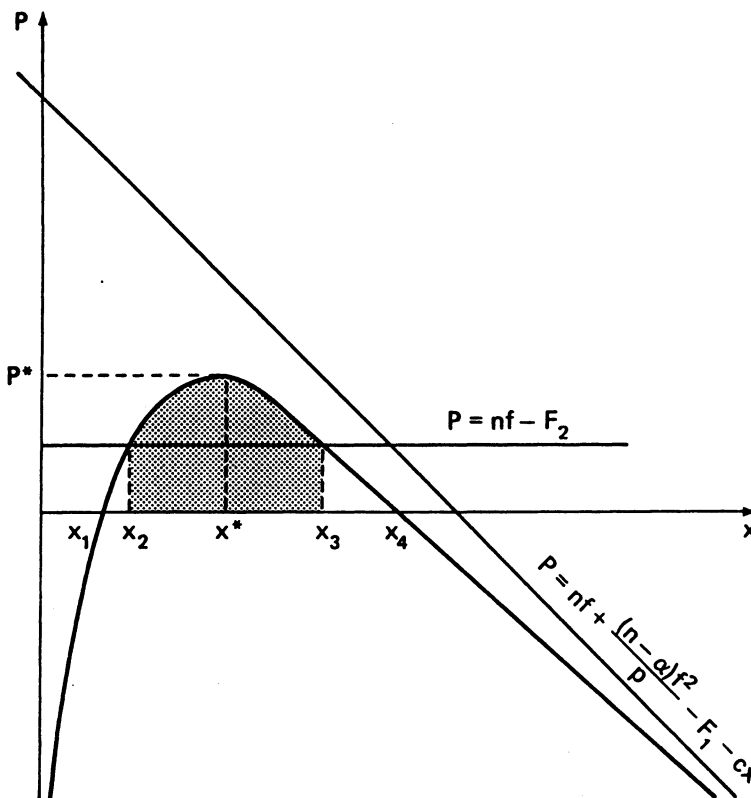


FIGURE 4

*Profit as a Function of Inspection Level for the Linear Case,  $\pi = kx$*

Hence  $\frac{\partial P^*}{\partial p} > 0$  when  $2 \left[ \frac{(n - \alpha)f^3 c}{k} \right]^{1/2} > (n - \alpha)f^2$  or  $f < \frac{4c}{k(n - \alpha)}$ .

Similarly  $\frac{\partial P^*}{\partial f} = n + \frac{2(n - \alpha)f}{p} - \frac{3}{p} \left[ \frac{(n - \alpha)cf}{k} \right]^{1/2}$ .

It follows that  $\frac{\partial P^*}{\partial f} > 0$  when  $\frac{3}{p} \left[ \frac{(n - \alpha)cf}{k} \right]^{1/2} < n + \frac{2(n - \alpha)f}{p}$ .

Hence  $\frac{\partial P^*}{\partial f} > 0$  certainly holds when  $f > \frac{c}{(n - \alpha)k}$ .

Thus  $\frac{\partial P^*}{\partial p} > 0, \frac{\partial P^*}{\partial f} > 0$  when  $\frac{c}{k(n - \alpha)} < f < \frac{4c}{k(n - \alpha)}$ .



This condition demonstrates that, provided  $f$  is suitably restricted, the maximum profit increases with an increase in both the fee and the penalty.

### CONCLUSION

In this paper we have constructed a model showing the relationship between public transport profits and levels of inspection. Once the profit curve has been generated, it is possible to locate

- (i) the range of inspection levels which generate positive profit;
- (ii) the profit-maximising inspection level;
- (iii) the region of the curve in which the authority needs to operate, in order to generate a profit greater than the profit which would be guaranteed if the population were totally honest.

The forms for  $\pi(x)$  and  $\pi_i(x)$  have been chosen primarily for mathematical tractability, but we do believe that they provide reasonably realistic first approximations. Moreover, substitution of feasible numerical values for the parameters in the model yields levels of inspection of passengers that exceed the apparent current levels of inspection employed by many transport operators. Empirical considerations will be of vital importance to further refinement of the model.

In the first instance, it will be of interest to find empirical data on an authority's actual policy-making procedure and the region of the profit graph in which it is operating (or thinks it is operating). There are definite policy implications in being located at different points along the inspection axis; the level of inspection ought to be consistent with the aim of the inspection operation.

Although not included in this article, a qualitatively similar analysis to the linear case applies for a non-linear  $\pi(x)$ , with consequential policy implications that are predictably similar. The fact that the profit function is dependent on the number of potential evaders who actually do evade, however, means that the distribution of  $\pi_i$  across individuals will affect this shape. The shape of this distribution has been assumed triangular; there is a need to investigate other distributions based on empirical evidence and their effect on the profit-vs-inspection level curve.

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